Intermodulation products

Multifrequency Lock-in Measurement

Intermodulation Products has developed a new kind of lock-in measurement instrument, designed to work with many frequencies, all locked to one reference oscillation.

The Lock-in technique

Signal-to-noise ratio is always improved when a reference oscillator is used to make a narrow-band measurement of some physical process[1]. An oscillator excites a physical system with a sinusoidal signal of a particular frequency. The reference oscillation, phase-locked to the excitation oscillation is used to demodulate, 'locking in' on the excitation frequency to determine the systems response to the excitation. A single-frequency lock-in measures two quadrature amplitudes: The response amplitude in-phase with the excitation I, and the quadrature amplitude Q, or that $\pi/2$ phase shifted with respect to the reference oscillation. These two quadratures give the Fourier cosine and Fourier sine components of the system response, at the frequency of the excitation.

Multifrequency lock-in

The lock-in concept can be generalized to multiple frequencies and the Multifrequency Lock-in Amplifier (MLA™) from Intermodulation Products is especially designed to handle many frequencies at the same time, or in the same measurement time T_m. The MLA™ will excite a physical system with a waveform consisting of many components at different frequencies, and measure both quadratures of the response at these many frequencies. We refer to the n components of the excitation signal and response signal as **tones**, each having a frequency f_n, and two quadratures of response I_n and Q_n.

The MLA™ is special in that all tones can be locked to *one reference oscillation*, which is fixed by an integer relation to the measurement bandwidth, or inverse of the measurement time $\Delta f = 1/T_m$. Unlike the multifrequency options of other lock-in instruments, which are simply multiple independent lock-ins in one box, the MLA™ uses special digital algorithms to achieve a synchronous mode of operation. The frequencies of the multiple tones are carefully optimized in a process called tuning, so that all frequencies can lock to the reference oscillation, in the given bandwidth.

The MLA™ generates a number of tones, as many as 42 can be chosen with frequency, amplitude and phase determined by the user. The user can choose to drive the system at any of these tones, or use the tone to measure only response at a frequency where there is no drive. Nonlinear systems respond at non-driven frequencies and the MLA™ is particularly powerful for probing nonlinear response.

Fourier leakage

The various tones of a multifrequency signal can affect each other in different ways. If the physical system that carries the signal is perfectly linear, the principle of superposition applies: The total response is simply the sum, or linear combination of the response at all tones. However, Fourier leakage may affect our ability to demodulate and measure the various tones independently. Fourier leakage occurs when a tone does not perform an integer number of oscillations in the measurement time window. A strong tone will leak over and destroy the measurement of other weaker tones that are close in frequency to the strong tone. Fourier leakage can be reduced by so-called windowing, or equivalently, low pass filtering of

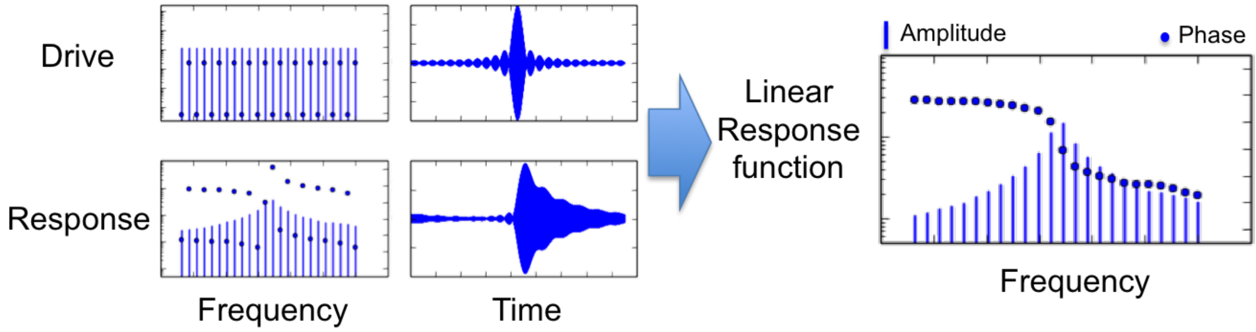

Figure 1: An AFM cantilever far from a surface is driven with a flat frequency comb consisting of 24 tones. In the time domain this comb represents a periodic train of sync pulses. The cantilevers response at each tone is captured by the MLA™. Dividing the response comb with the drive comb gives the linear response function of the cantilever. The resonance curve of the cantilever is measured without sweeping the frequency. When the cantilever is oscillating close to a surface where nonlinearity results in strong intermodulation between drive tones, simple division to get linear response is no longer valid. However, other methods exist for extracting the nonlinearity, or tip-surface forces from the multifrequency response.

the demodulated signal, but these methods are imperfect and they can not completely prevent leakage.

To completely avoid Fourier leakage, the MLA™ is designed to work only with frequencies that do perform an integer number of oscillations in the measurement time window. Before measurement one applies a tuning function that takes the user-defined target frequencies and returns the best choice of frequencies which will not leak in to one-another. This process is in direct analogy to the tuning of musical instruments so they can play together in the same key. The easiest way to tune the MLA™ is to start with a desired measurement bandwidth and then choose all frequencies to be integer multiples of the bandwidth $f_n = k_n \Delta f$, where k_n are a set of integers.

Because the MLA™ completely eliminates Fourier leakage, you can make a very efficient multi-frequency measurement of a linear response function. For example an Atomic Force Microscope (AFM) cantilever when it is freely oscillating, away from a surface (see fig. 1). All tones are measured at once and there is no need to sweep a single drive frequency through the resonance.

Nonlinear Response

Fourier leakage is an issue for the measurement of linear systems, but more fascinating is an effect that occurs in nonlinear systems, such as an AFM cantilever oscillating close to a surface[2]. A nonlinear system will generate new tones in the response waveform that are not in the drive waveform. Harmonics are generated at exact integer multiples of a drive frequency and they will occur in nonlinear systems driven with only one tone. Intermodulation products, or mixing products occur when two or more tones are present in the drive, and these occur at integer linear combinations of the drive frequencies. When many drive tones are present: $f_1, f_2, f_3 \cdots$, intermodulation response is generated at very many new frequencies,

$$f_{IMP} = k_1 f_1 + k_2 f_2 + k_3 f_3 + \cdots$$

were $k_1, k_2, k_3 \cdots$ are any integer.

One of the primary design goals of the MLA™ is the accurate measurement of many intermodulation products[3]. When the MLA™ is tuned, all frequencies in the nonlinear response will be at integer multiples of the bandwidth. Thus, tuning the measurement makes it possible to lock-in on the nonlinear response, which is also not affected by Fourier leakage.

References

1) Measurement of Thermal Radiation at Microwave Frequencies. R. H. Dicke Rev. Sci. Instrum. **17**, 268 (1946).

2) Intermodulation atomic force microscopy. D. Platz, E. A. Tholen, D. Pessen and D. B. Haviland. Appl. Phys. Lett. **92**, 153106 (2008).

3) The intermodulation lockin analyzer. E. A. Tholen, D. Platz, D. Forchheimer, V. Schuler, M. O. Tholen, C. Hutter and D. B. Haviland. Rev. Sci. Instr. **82**, 026109 (2011).